MATH-343 Complex Analysis

Credit Hours: 3-0

Prerequisites: None

Course Objectives: Complex variables is an important area from a purely mathematical point of view, as well as a powerful tool for solving a wide Varity of applied problems. It is founds its applications in many mathematical disciplines, including in particular real analysis, differential equations, algebra and topology. This course develops the theory of functions of a complex variable, emphasizing their geometric properties and some applications. It also treats the traditional theorems, algorithms, and applications of complex analysis. These include: finding of complex roots for polynomial equations and complex integration, residue theory and its applications.

Core contents: Complex Numbers, Analytic Functions, Elementary Functions, Integrals, Series, Residues and Poles, Conformal Mapping

Detailed Course Contents: Complex Numbers: Sums and Products, Basic Algebraic Properties, Further Properties , Vectors and Moduli , Complex Conjugates , Exponential Form , Products and Powers in Exponential Form , Arguments of Products and Quotients , Roots of Complex Numbers , Examples, Regions in the Complex Plane.

Analytic Functions: Functions of a Complex Variable, Mappings, Mappings by the Exponential Function, Limits, Theorems on Limits, contents, Limits Involving the Point at Infinity, Continuity, Derivatives, Differentiation Formulas, Cauchy–Riemann Equations, Sufficient Conditions for Differentiability, Polar Coordinates, Analytic Functions, Examples, Harmonic Functions, Uniquely Determined Analytic Functions, Reflection Principle.

Elementary Functions: The Exponential Function, The Logarithmic Function, Branches and Derivatives of Logarithms, Some Identities Involving Logarithms, Complex Exponents, Trigonometric Functions, Hyperbolic Functions, Inverse Trigonometric and Hyperbolic Functions.

Integrals: Derivatives of Functions w(t), Definite Integrals of Functions w(t), Contours, Contour Integrals, Some Examples, Examples with Branch Cuts, Upper Bounds for Moduli of Contour Integrals, Antiderivatives, Cauchy–Goursat Theorem, Simply Connected Domains, Multiply Connected Domains, Cauchy Integral Formula, An Extension of the Cauchy Integral Formula, Some Consequences of the Extension, Liouville's Theorem and the Fundamental Theorem of Algebra, Maximum Modulus Principle

Series: Convergence of Sequences, Convergence of Series, Taylor Series, Laurent Series, Absolute and Uniform Convergence of Power Series, Continuity of Sums of Power Series, Integration and Differentiation of Power Series, Uniqueness of Series Representations, Multiplication and Division of Power Series.

Residues and Poles: Isolated Singular Points, Residues, Cauchy's Residue Theorem, Residue at Infinity, The Three Types of Isolated Singular Points, Residues at Poles, Zeros of Analytic Functions, Zeros and Poles, Behavior of Functions Near Isolated Singular Points

Applications of Residues: Evaluation of Improper Integrals, Improper Integrals from Fourier Analysis, Indented Paths, Definite Integrals Involving Sines and Cosines.

Course outcomes: Students are expected to understand:

- The complex number and their geometric interpretation.
- Functions of a complex variable, limits, continuity.
- The all-important concepts of the derivative of a complex function and analyticity of afunction.
- The trigonometric, exponential, hyperbolic, and logarithmic Functions.
- The famous Cauchy-Goursat theorem and the Cauchy integral formulas.
- Concepts of complex sequences and infinite series and the Laurent series, residues, andthe residue theorem and its applications

Text Book: James W. Brown and R.V. Churchill, Complex Variables and Applications, 8th ed.,McGraw-Hill, 2009.

Reference Books

1. Fundamentals of Complex Analysis, 3rd Edition, E.B. Saff and Arthur D. Snider.Prentice Hall, 2003.
2. Visual Complex Analysis, Tristan Needham, Oxford University Press, 1997.
3. Dennis G. Zill A First Course In Complex Analysis With Applications, 2003 by Jonesand Bartlett Publishers.

Weekly Breakdown		
Week	***Section***	***Topics***
1	1-13	Sums and Products, Basic Algebraic Properties, Further Properties, Vectors and Moduli, Complex Conjugates.
2	16-34	Exponential Form, Products and Powers in Exponential Form, Arguments of Products and Quotients, Roots of Complex Numbers, Regions in the Complex Plane.
3	35-52	Functions of a Complex Variable, Mappings, Mappings by the Exponential Function, Limits, Theorems on Limits, contents, Limits Involving the Point at Infinity.
4	53-88	Continuity, Derivatives, Differentiation Formulas, Cauchy–Riemann Equations, Sufficient Conditions for Differentiability, Polar Coordinates, Analytic Functions, Examples, Harmonic Functions, Uniquely Determined Analytic Functions, Reflection Principle.
5	89-100	The Exponential Function, The Logarithmic Function, Branches and Derivatives of Logarithms, Some Identities Involving Logarithms.
6	101-116	Complex Exponents, Trigonometric Functions, Hyperbolic Functions, Inverse Trigonometric and Hyperbolic Functions.
7	117-126	Derivatives of Functions w(t), Definite Integrals of Functions w(t), Contours
8	127-145	Contour Integrals, Some Examples, Examples with Branch Cuts, Upper Bounds for Moduli of Contour Integrals, Antiderivatives.

9	**Mid Semester Exam**	
10	150-175	Cauchy–Goursat Theorem, Simply Connected Domains, Multiply Connected Domains, Cauchy Integral Formula, An Extension of the Cauchy Integral Formula, Some Consequences of the Extension, Liouville's Theorem, Fundamental Theorem of Algebra and Maximum Modulus Principle(without proof)
11	181-209	Convergence of Sequences and Series, Taylor Series, Laurent Series, Absolute and Uniform Convergence of Power Series,
12	211-228	Continuity of Sums of Power series, Integration and Differentiation of Power Series, Uniqueness of Series Representations, Multiplication and Division of Power Series.
13	229-243	Isolated Singular Points, Residues, Cauchy's Residue Theorem, Residue at Infinity, The Three Types of Isolated Singular Points
14	244-257	Residues at Poles, Zeros of Analytic Functions, Zeros and Poles, Behavior of Functions Near Isolated Singular Points.
15	261-279	Evaluation of Improper Integrals, Improper Integrals from Fourier Analysis, Indented Paths
16	288-290	Definite Integrals Involving Sines and Cosines
17		Review
18	**End Semester Exam**	